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Using the MPFRL-WASPAS Model for the Selection of AI-Based Early Warning Systems Based on Natural Disaster Management and Frank Aggregation Operators

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ABSTRACT

The selection or assessment of the artificial intelligence-based early warning systems for natural disaster management is very valuable and critical because the selection of the right system directly affects how effectively, quickly, and accurately disasters can be communicated and detected to at-risk communities. The main theme of this manuscript is to develop the novel system of multi-polar fuzzy rough linguistic sets and also describe their valuable operational laws. Further, this study also concentrates on the valuation of the multi-polar fuzzy rough linguistic weighted aggregated sum product assessment models based on Frank norms. Therefore, the averaging operator and geometric operator based on Frank operational laws are also developed for the construction of the above models. Moreover, this study also illustrates some numerical examples for the evaluation of the best and worst decisions among the collection of the considered data. Ultimately, choosing an appropriate artificial intelligence-based early warning system helps reduce loss of life, enhance overall disaster preparedness and resilience, and minimize economic damage. Finally, we compare the ranking values of the proposed information with the ranking values of the existing systems to describe the validity of the invented approaches.

a. Introduction

Natural disasters are one of the most emerging and critical challenges faced by communities around the world. These instant events, including earthquakes, landslides, cyclones, wildfires, and floods damages human life and cause a huge economic disruption every year. The intensity of natural Disasters is increasing day by day because of environmental degradation, rapid urbanization, and climate change. That is why it becomes a very critical concern for policymakers, governments, and all concerned authorities and organizations. In this context, early warning systems play a crucial role because it helps to reduce the impact of natural disasters. These systems are specially designed to detect disasters earlier, before they occur. It gives early, timely alerts to communities and all responsible authorities, which enables people to take preventive actions such as emergency planning, infrastructure protection, and evacuation.

An effective and well-organized early detection system can minimize both the human and economic losses. Nowadays, artificial intelligence (AI) enhances the efficiency and effectiveness of the early warning systems. AI enables these systems to effectively handle large amounts of data and accurately analyze each pattern. AI completely revolutionized the approach of early warning systems and made it reliable for any complex situation. The key advantages of AI-based early detection systems are briefly explained in Table 1 below.

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Table 1: Key advantage of AI-based early warning systems.

Advantages	Explanation	Impact
Faster Response Time	AI-based early warning systems continuously monitor the system, which allows quick fault detection.	This approach helps to reduce system downtime and enhance operational processes.
Scalability	All these AI-based early warning systems effectively handle large and complex power networks.	This approach helps to enhance the adoptability of the systems.
Improved Accuracy	AI enables these early warning systems to detect faults more accurately compared to traditional systems.	This approach reduces false alarms and outages.
Data Integration	It enables the systems to effectively integrate data from multiple sources and sensors.	This approach improves the accuracy of fault diagnosis.
Predictive Maintenance	AI provides early signs of failures before they occur.	It supports the continuous operational process and avoids sudden breaks.

Although AI-based early warning systems effectively handle complex situations and help to reduce losses. Despite all these, there are some limitations or challenges faced by AI-based early warning systems. Each early warning system has its own effectiveness and limitations, and all of them perform differently from one another. Some systems may give high accuracy but require high initial cost, while others may require less initial cost but are less reliable. Similarly, some approaches may effectively monitor the operational process but require a huge amount of time, while some take less time to operate but have a data integration challenge. So, all these differences make the evaluation and selection process of an effective system very complex. Some key challenges of AI-based early warning systems are briefly discussed in Table 2 below:

Table 2: Key Challenges of AI-based early warning systems.

Challanges	Explination	Impact
High Implementation Cost	These systems require a high implementation cost because the AI devices are highly expensive.	This approach reduces the adoptability of these systems and makes them unaffordable for many regions.
Model Interpretability	The structure of AI-based early warning systems is so complex that it is very difficult to explain to stakeholders.	This approach reduces trust and acceptance of the system.
Data Quality and Availability	These systems completely depend on the data, and incomplete or noisy data can reduce the accuracy of the system.	This approach reduces the reliability of the warnings of the systems.
Response Time	Sometimes, dealing with complex data reduces alert issuances because of slow processing.	It affects the timely disaster response and reduces system efficiency.
Integration Complexity	The integration of data sources is very challenging.	It also affects the efficiency of the overall system.

Before adopting any systems, it is very important for the decision maker to understand both the advantages and limitations of these systems. Without understanding it, is very crucial to get an accurate and reliable AI-based early warning system. In this context, the scholars invented different early warning approaches, such as Charan et al. [1] constructed a novel early warning system using wireless sensor networks for detecting and mitigating natural disasters. Alphonsa and Ravi [2] invented a strong early warning system for earthquakes using the concept of the Internet of Things. Ali et al. [3] constructed a novel prediction approach using an artificial neural network approach for cyclone tracks in the north Indian Ocean. Sahoo and Bhaskaran [4] invented an artificial neural network-based predicting approach for storm surge and coastal inundation. Akhyar et al. [5] discussed applications of deep artificial intelligence for natural disaster management systems.

Decision-making is to select the most appropriate option from the available choices while considering multiple criteria. In most real-world situations, the decision-makers often deal with uncertain and incomplete information. In this context, the traditional decision-making approaches are limited to addressing such information because they completely rely on precise numerical values. They are limited to addressing the subjective expert preferences. To bridge this gap, Zadeh [6] constructed the idea of a fuzzy set (FS), which is a generalization of the existing classical set. FS effectively captures ambiguous and uncertain information by assigning a partial membership degree to each element of the set. This partial membership degree belongs from close interval of 0 and 1. The integrated approach of FS and decision-making models enables the decision makers to effectively model uncertainty. These fuzzy-based decision-making approaches provide more reliable and flexible solutions. The research scholars widely applied these models to handle complex real-world problems across multiple domains, such as Wieckowski et al. [7] invented a novel ranking approach under a fuzzy environment for aggregating results from multi-criteria assessment, and to address the challenges faced by the traditional aggregation techniques. Dagdeviren and Yuksel [8] constructed a novel AHP (analytic hierarchy process) approach under a fuzzy environment for behavior-based safety management. Ibraheem and Hamad [9] invented a combined framework of fuzzy approach and association rule mining and discussed its applications in e-commerce. Abdalla et al. [10] presented a hybrid approach by integrating the idea of the Internet of Things with a fuzzy approach and extended its application to the medical domain. Krechko and Mikhaylov [11] used the integrated approach of spatial analysis and FS to handle global electricity generation from renewable sources. Sharma et al. [12] combined the ideas of Internet of Things-based block chain with a fuzzy approach for enhancing smart grid efficiency.

The FS theory effectively handles uncertainty and vagueness by assigning a partial membership function. This approach is highly reliable for situations where the information came from a single source. It is limited to handling such information that comes from multiple sources. To overcome this limitation, Chen et al. [13] constructed the novel notation of multi-polar fuzzy set (m-PFS), which is a generalization of the traditional FS. The m-PFS captures uncertainty and vagueness by assigning multiple membership functions to each element of the set instead of a single one. The m-PFS effectively captures complex and uncertain expert information. It can easily integrate with decision-making models and enhance the evaluation approaches. The m-PFS framework is widely extended by researchers to different domains to handle real-world problems. Ali and Alsager [14] constructed a hybrid framework by integrating Heronian mean with power aggregation operators under an m-polar fuzzy domain for urban transportation management. Jagtap and Ghangale [15] invented a novel TOPSIS (Technique for Ordered Preferences by Similarity to Ideal Solutions) approach under an m-polar fuzzy environment for the selection of electrical vehicle. Ali et al. [16] constructed some novel aggregation approaches under an m-polar fuzzy environment and discussed their applications in multi-criteria decision-making (MCDM). Akram et al. [17] constructed connectivity indices of a network approach based on m-polar fuzzy information and extended its applications to the product manufacturing environment. Jagtap and Karande [18] established a novel m-polar fuzzy-based algorithm and discussed its applications in non-traditional machining process selection. Jagtap and Karande [19] presented a detailed review of the evolution of m-PFS as a decision-making tool. Rahman et al. [20] invented a generalized notation of Aczel-Alsina aggregation operators based on m-polar fuzzy information and extended their applications to wind power plant sites.

The m-PFS effectively models the uncertainty by assigning multiple membership values to represent different attributes of a decision. It is reliable for the problems where uncertainty arises only from vagueness. However, in many real-world decision-making problems, the uncertainty also arises from incomplete information. The m-PFS is limited to modeling this situation. To overcome this limitation, Pawlak [21] invented the notation of Rough Set (RS). RS effectively handles such problems using the idea of lower and upper approximation based on given information. RS only handles the uncertainty that arises from incomplete information, but many decision-making problems, however, the information is not just insufficient but also vague. To handle such a situation, Dubois and Prade [22] constructed a combined approach of Rough Fuzzy Set (RFS) and Fuzzy Rough Set (FRS) by integrating the existing ideas of FS and RS. This integrated approach enables the decision makers to model complex decision-making problems effectively. This integrated approach is widely used across different domains such as Ahmed et al. [23] invented a novel fitting approach for FS based information system under a fuzzy rough environment. Ji et al. [24] constructed a novel neural network approach under a fuzzy rough environment and extended its applications for feature selection. Eldakhly [25] invented an integrated framework of deep learning and fuzzy rough approach for early detection of ovarian cancer. Hancer [26] constructed a hybrid framework of differential evaluation and fuzzy rough approach for constructing a filter approach for feature selection. Zhou et al. [27] invented a novel data mining approach using the combined approach of FRS and fuzzy neural network. Wang et al. [28] invented a strong, sustainable approach under a fuzzy rough environment for energy-efficient smart city development.

All these ideas are completely relying on numerical membership values. However, in most real-world decision-making scenarios, the expert prefers to express their judgment using linguistic terms such as very low, low, medium, high, and very high rather than a fixed numerical value. To overcome this limitation, Zadeh [29] invented the idea of Linguistic Set (LS). LS enable the expert to express their preferences naturally. It provides more clear representation of the expert information by using linguistic terms. These terms can be mathematically modeled to enhance computation while preserving the real meaning of the expert information. Scholars widely used the linguistic concept across multiple domains for handling real-world decision-making problems. For example, Adamopoulos and Pappis [30] invented a novel fuzzy linguistic approach for handling multi-criteria sequential problems. Jeon et al. [31] constructed a fuzzy assembly approach under a linguistic environment and extended its applications to an interlaced HDTV sequence environment. Ren and Hao [32] invented a novel fusion approach of fuzzy numbers and linguistic terms under individual semantics in decision-making problems. Arfi [33] invented a novel fuzzy approach based on linguistic information for decision-making in politics. Herrera-Viedma et al. [34] constructed a hybrid model using fuzzy and linguistic approaches under multi-granular linguistic information. Merigo and Casanovas [35] invented some novel aggregation approaches under a linguistic environment and extend its applications to a decision-making environment. Umano et al. [36] invented linguistic labels for the representation of fuzzy preference relations under fuzzy group decision making. Xu et al. [37] constructed the idea of power aggregation operators based on linguistic information for Multi-Attribute Group Decision-Making (MAGDM) problems.

These individual ideas effectively modeled uncertain and complex expert information across different fields. Due to its effectiveness, the scholars constructed many hybrid ideas by integrating these individual ideas into one frame, such as Adeel et al. [38] invented the idea of a multi-Polar Fuzzy Linguistic (m-PFL) framework by integrating the existing idea of m-PFS and LS.Agarwal and Palpanas [39] constructed the idea of linguistic rough set (LRS) by combining the existing concepts of LS and RS. Based on these individual and hybrid ideas, the researchers invented multiple aggregation approaches to aggregate expert information effectively, such as Akram et al. [40] invented a modified notation of Dombi aggregation operators under an m-polar fuzzy environment for decision-making problems. Maity and Pal [41] invented a generalized idea of Dombi power aggregation operators based on m-polar fuzzy information and extended its applications to a Multi-Attribute Decision-Making environment (MADM). Waseem et al. [42] constructed an extended notation of Hamacher aggregation operators under m-polar fuzzy information for MADM problems. Deveci et al. [43] invented a modified Hamacher aggregation operator under a fuzzy environment for the evaluation of climate change. Among all existing aggregation approaches, the Frank aggregation operators, constructed by Frank [44] using Frank t-norm (FTN) and Frank t-conorm (FTCN) concept achieves wide attention because of their robust and flexible structure. Later on, Ye [45] generalized the traditional idea of Frank aggregation operators to the fuzzy domain for the impact of massive open online courses in higher education. Similarly, based on this information, the scholars constructed different MADM approaches for the evaluation and ranking of alternatives based on multiple conflicting criteria. For example, Chakraborty and Zavadskas [46] proposed a novel MADM approach named WASPAS (Weighted Aggregated Sum Product Assessment) model for manufacturing decision making. Turskis et al. [47] combined the existing ideas of the Analytic Hierarchy Process and WASPAS approach under a fuzzy domain for selecting a construction site. Ahmmad et al. [48] constructed a generalized notation of the MABAC approach under a fuzzy rough environment for disability support systems. Kizielewicz and Baczkiewicz [49] integrated different MADM approaches under a fuzzy environment for the housing selection problem. Akram et al. [50] invented a hybrid approach by combining the PROMETHEE model and Analytic Hierarchy Process under an m-polar fuzzy environment for group decision-making problems.

These aggregation approaches and MADM models are widely applied across multiple domains to evaluate and rank alternatives under complex situations. It enables the decision makers to address real-world decision-making problems effectively. Despite the effectiveness of these ideas, there are still huge research gaps in the existing literature that need to be bridged. Some key gaps are as follows:

1. The current literature has many hybrid ideas that help the expert to express their preferences, but the hybrid multi-Polar Fuzzy Rough Linguistic (m-PFRL) framework has not developed yet.
2. The existing literature contains many aggregation ideas, but the concept of Frank aggregation operators based on m-PFRL information has not developed yet.
3. The current literature offers different evaluation approaches for the alternatives, but is limited to offer evaluation approach for m-PFRL information.
4. Many different generalized structures of the WASPAS approach have been developed, but the WASPAS approach based on m-PFRL information has not constructed yet.

The motivation of this study came from the above-mentioned research gaps. To cover them, in this study, we aim to develop some novel and effective ideas. The following are our central targeted ideas that will cover these research gaps.

1. Our focus is to integrate the existing ideas of FS, m-PFS, RS, and LS into one frame and invent a novel m-PFTL framework from it.
2. Our target is to extend the existing notation of Frank aggregation operators to the m-PFRL environment.
3. We aim to investigate an evaluation approach that effectively handles the expert information expressed in m-PFRL form.
4. We also aim to modify the existing approach of the WASPAS model to enhance its evaluation and ranking accuracy.

The hybrid approaches can effectively handle the expert preferences expressed in complex form. It helps the decision makers to model uncertain and ambiguous information effectively.

The key advantages of the proposed work are briefly explained below:

1. The hybrid m-PFRL approach enables the expert to express their preferences in a generalized way, even if it is complex.
2. The advanced Frank aggregation operators provide an effective framework for the aggregation of alternatives.
3. The generalized WASPAS model based on m-PFRL information enhances the evaluation and ranking mechanism of the alternatives and enables the decision makers to easily get the best among the selected ones.
4. The operational mechanism of the generalized WASPAS approach is very clear and flexible, and can easily be extended to multiple domains.

The m-PFRL framework combines the effectiveness and strengths of many individual ideas, such as FS, m-PFS, RS, and LS. That is why it is a generalization of all these ideas, and these have now become its special cases. Similarly, the generalized Frank aggregation operators and WASPAS model are also extensions of multiple existing ideas, such as fuzzy based frank aggregation operators, m-polar fuzzy based frank aggregation operators, rough based frank aggregation operators, WASPAS approach based on fuzzy information, WASPAS approach based on m-polar fuzzy information, and fuzzy rough-based WASPAS approach, and all these current ideas have now become their special cases. In this article, our focus is to establish the following ideas:

1. To construct the novel idea of the m-PFRL set and defines its fundamental properties.
2. To extend the existing idea of Frank aggregation operators to the m-PFRL environment.
3. To generalize the existing notation of the WASPAS approach and invent a novel WASPAS approach based on m-PFRL information.
4. To address a real-world problem of AI-based early warning systems for natural disaster management using the proposed idea.
5. To compare the results of the proposed idea with current related ideas to check its accuracy and reliability.

In this manuscript, the information is arranged in the following way. In Section 2, we briefly revised the current idea of FS, m-PFS, RS, and LS. In Section 3, we construct the novel idea of the m-PFRL set and explain its fundamental operational laws. In Section 4, we explained the novel idea of the WASPAS approach based on m-PFRL information. In Section 5, we briefly explained a real-world problem of AI-based early warning systems for natural disaster management and addressed it with the help of a numerical example. In Section 6, we compared the results of the proposed approach with some current ideas, and in Section 7, we put the concluding remarks.

b. Preliminaries

In this section, we revised the concept of a well-known FTN family. We also revised some existing ideas, such as FS and LS. RS and m-PFS. We briefly defined the fundamental operational laws of FS and m-PFS. This section helps the reviewers to understand the background of this research work.

Definition 1: [44] The concept of FTN and FTCN was invented by M.J. Frank in 1970. It is a well-known t-norm family and is defined as:

$$\mathfrak{T}_{\mathfrak{I}}^{\mathfrak{F}}(\mathfrak{a},\mathfrak{b}) = \begin{cases} \mathfrak{T}_{min}(\mathfrak{a},\mathfrak{b}) & \\ \mathfrak{T}_{prod}(\mathfrak{a},\mathfrak{b}) = \mathfrak{a},\mathfrak{b} & if\ \mathfrak{I}=0 \\ \mathfrak{T}_{Luk}(\mathfrak{a},\mathfrak{b}) = \max\{0,\mathfrak{a}+\mathfrak{b}-1\} & if\ \mathfrak{I}=1 \\ \log_{\mathfrak{I}}\left(1+\dfrac{(\mathfrak{I}^{\mathfrak{a}}-1)(\mathfrak{I}^{\mathfrak{b}}-1)}{\mathfrak{I}-1}\right) & if\ \mathfrak{I}=+\infty \\ & Otherwise \end{cases}$$

The additive generator of this well-known t-norm family is defined as

$$\mathcal{F}_{\mathfrak{I}}^{\mathfrak{F}}(\mathfrak{a}) = \begin{cases} -\log\mathfrak{a} & \text{if } \mathfrak{I}=0 \\ 1-\mathfrak{a} & \text{if } \mathfrak{I}=+\infty \\ \log\dfrac{\mathfrak{I}-1}{\mathfrak{I}^{\mathfrak{a}}-1} & Otherwise \end{cases}$$

In this research work, we will use the following FTN and FTCN, respectively

$$\mathfrak{T}_{\mathfrak{I}}^{\mathfrak{F}}(\mathfrak{a},\mathfrak{b}) = \log_{\mathfrak{I}}\left(1+\frac{(\mathfrak{I}^{\mathfrak{a}}-1)(\mathfrak{I}^{\mathfrak{b}}-1)}{\mathfrak{I}-1}\right)$$

Definition 2: [6] Assume that $\mathfrak{K}^{\mathfrak{U}_{a}}$ be a universal set of discourse such that $\mathfrak{K}^{\mathfrak{U}_{a}} = (\mathfrak{K}^{u_1},\mathfrak{K}^{u_2},\mathfrak{K}^{u_3},\dots,\mathfrak{K}^{u_n})$. Then, the FS is defined as:

$$\mathfrak{K}^{\mathbb{F}_s} = \left\{\left(\mathfrak{K}^{u_n},\left(\mathfrak{K}^{\mathfrak{MD}}(\mathfrak{K}^{u_n})\right)\right) | \mathfrak{K}^{u_n} \in \mathfrak{K}^{\mathfrak{U}_a}\right\}$$

Where the term$\mathfrak{K}^{\mathfrak{MD}}(\mathfrak{K}^{u_n})$shows the membership degree, which fulfils the conditions that $\mathfrak{K}^{\mathfrak{MD}}(\mathfrak{K}^{u_n}) \in [0,1]$. a fuzzy number (FN) can be written as $\mathfrak{K}_{\mathbb{N}}^{\mathbb{F}_s} = (\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}})$. now, assume $\mathfrak{K}_1^{\mathbb{F}_s} = \mathfrak{K}_1^{\mathfrak{MD}}$, and $\mathfrak{K}_2^{\mathbb{F}_s} = \mathfrak{K}_2^{\mathfrak{MD}}$ be two FN. Then, for all FN, the fundamental operational laws are defined as

$$\mathfrak{K}_1^{\mathbb{F}_s} \oplus \mathfrak{K}_2^{\mathbb{F}_s} = \left(\mathfrak{K}_1^{\mathfrak{MD}} + \mathfrak{K}_2^{\mathfrak{MD}} - \mathfrak{K}_1^{\mathfrak{MD}}\mathfrak{K}_2^{\mathfrak{MD}}\right)$$

$$\mathfrak{K}_1^{\mathbb{F}_s} \otimes \mathfrak{K}_2^{\mathbb{F}_s} = \left(\mathfrak{K}_1^{\mathfrak{MD}}\mathfrak{K}_2^{\mathfrak{MD}}\right)$$

$$\lambda\mathfrak{K}_1^{\mathbb{F}_s} = \left(1-\left(1-\mathfrak{K}_1^{\mathfrak{MD}}\right)^{\lambda}\right)$$

$$\left(\mathfrak{K}_1^{\mathbb{F}_s}\right)^{\lambda} = \left(\mathfrak{K}_1^{\mathfrak{MD}}\right)^{\lambda}$$

Definition 3: [29] assume that a finite and ordered linguistic set is shown by $\mathfrak{K}^{\mathfrak{L}} = \{\mathfrak{K}^{\mathbb{l}_1},\mathfrak{K}^{\mathbb{l}_2},\mathfrak{K}^{\mathbb{l}_3}\dots,\mathfrak{K}^{\mathbb{l}_n}\}$, such that $\mathfrak{K}^{\mathbb{l}_i} = (i=1,2,3,\dots,n)$ represents linguistic terms. These linguistic terms show the qualitative expert information under the condition that

$$\mathfrak{K}^{\mathbb{l}_1} < \mathfrak{K}^{\mathbb{l}_2} < \mathfrak{K}^{\mathbb{l}_3} < \cdots < \mathfrak{K}^{\mathbb{l}_n}$$

Definition 4: [21] Let us assume $\mathfrak{K}_{\mathcal{A}} = (\mathfrak{U},\mathfrak{B})$ represents an approximation space such that a rough approximation $\mathfrak{K}^{\mathbb{R}}$ is defined as $\mathfrak{K}^{\mathbb{R}} = \mathfrak{K}_{\mathcal{A}}(\mathfrak{U}) \longrightarrow \mathfrak{K}_{\mathcal{A}}(\mathfrak{U}) \times \mathfrak{K}_{\mathcal{A}}(\mathfrak{U})$.

Then, for all $\mathfrak{K}^{ap} \in \mathfrak{K}_{\mathcal{A}}(\mathfrak{U})$, we have

$$\mathfrak{K}^{\mathbb{R}}(\mathfrak{K}^{ap}) = \left(\underline{\mathfrak{K}^{ap}},\overline{\mathfrak{K}^{ap}}\right)$$

Where $\underline{\mathfrak{K}^{ap}}$ show the lower approximation and $\overline{\mathfrak{K}^{ap}}$ shows the upper approximation and is defined as

$$\underline{\mathfrak{K}^{ap}} = \{z \in \mathfrak{U}: [z]_{\mathfrak{K}^{\mathbb{R}}} \subseteq \mathfrak{K}^{ap}\}$$

$$\overline{\mathfrak{K}^{ap}} = \{z \in \mathfrak{U}: [z]_{\mathfrak{K}^{\mathbb{R}}} \cap \mathfrak{K}^{ap} \neq \emptyset\}$$

Then, $(\underline{\mathfrak{K}^{ap}},\overline{\mathfrak{K}^{ap}})$ is called RS, if $\underline{\mathfrak{K}^{ap}} \neq \overline{\mathfrak{K}^{ap}}$

Definition 5: **[13]** Let us consider $\mathfrak{K}^{\mathfrak{U}_a}$ shows a universal set of discourse such that $\mathfrak{K}^{\mathfrak{U}_a} = (\mathfrak{K}^{u_1},\mathfrak{K}^{u_2},\mathfrak{K}^{u_3},\dots,\mathfrak{K}^{u_n})$
Then, an m-PFS is defined as

$$\mathfrak{K}^{\mathcal{M}_p} = \left\{\left(\mathfrak{K}^{u_n},\left(\mathfrak{K}^{\mathfrak{MD}_1}(\mathfrak{K}^{u_n}),\mathfrak{K}^{\mathfrak{MD}_2}(\mathfrak{K}^{u_n}),\mathfrak{K}^{\mathfrak{MD}_3}(\mathfrak{K}^{u_n}),\dots,\mathfrak{K}^{\mathfrak{MD}_m}(\mathfrak{K}^{u_n})\right)\right) | \mathfrak{K}^{u_n} \in \mathfrak{K}^{\mathfrak{U}_a}\right\}$$

Where $\left(\mathfrak{K}^{\mathfrak{MD}_1}(\mathfrak{K}^{u_n}),\mathfrak{K}^{\mathfrak{MD}_2}(\mathfrak{K}^{u_n}),\mathfrak{K}^{\mathfrak{MD}_3}(\mathfrak{K}^{u_n}),\dots,\mathfrak{K}^{\mathfrak{MD}_m}(\mathfrak{K}^{u_n})\right)$ is a family of membership degrees that fulfil the condition that

$$\left(\mathfrak{K}^{\mathfrak{MD}_1}(\mathfrak{K}^{u_n}),\mathfrak{K}^{\mathfrak{MD}_2}(\mathfrak{K}^{u_n}),\mathfrak{K}^{\mathfrak{MD}_3}(\mathfrak{K}^{u_n}),\dots,\mathfrak{K}^{\mathfrak{MD}_m}(\mathfrak{K}^{u_n})\right) \in [0,1].$$

An m-polar fuzzy number (m-PFN) is represented by

$$\mathfrak{K}_{\mathbb{N}}^{\mathcal{M}_p} = \left(\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_1},\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_2},\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_3},\dots,\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_m}\right).$$

Now, consider

$$\mathfrak{K}_1^{\mathcal{M}_p} = \left(\mathfrak{K}_1^{\mathfrak{MD}_1},\mathfrak{K}_1^{\mathfrak{MD}_2},\mathfrak{K}_1^{\mathfrak{MD}_3},\dots,\mathfrak{K}_1^{\mathfrak{MD}_m}\right)$$

And $\mathfrak{K}_2^{\mathcal{M}_p} = \left(\mathfrak{K}_2^{\mathfrak{MD}_1},\mathfrak{K}_2^{\mathfrak{MD}_2},\mathfrak{K}_2^{\mathfrak{MD}_3},\dots,\mathfrak{K}_2^{\mathfrak{MD}_m}\right)$ be two m-PFN.

Then, for all m-PFN, the basic operational laws are constructed as

$$\mathfrak{K}_1^{\mathcal{M}_p} \oplus \mathfrak{K}_1^{\mathcal{M}_p} = \left(\mathfrak{K}_1^{\mathfrak{MD}_1} + \mathfrak{K}_2^{\mathfrak{MD}_1} - \mathfrak{K}_1^{\mathfrak{MD}_1}\mathfrak{K}_2^{\mathfrak{MD}_1}, \mathfrak{K}_1^{\mathfrak{MD}_2} + \mathfrak{K}_2^{\mathfrak{MD}_2} - \mathfrak{K}_1^{\mathfrak{MD}_2}\mathfrak{K}_2^{\mathfrak{MD}_2},\dots,\mathfrak{K}_1^{\mathfrak{MD}_m} + \mathfrak{K}_2^{\mathfrak{MD}_m} - \mathfrak{K}_1^{\mathfrak{MD}_m}\mathfrak{K}_2^{\mathfrak{MD}_m}\right)$$

$$\mathfrak{K}_1^{\mathcal{M}_p} \otimes \mathfrak{K}_1^{\mathcal{M}_p} = \left(\mathfrak{K}_1^{\mathfrak{MD}_1}\mathfrak{K}_2^{\mathfrak{MD}_1},\mathfrak{K}_1^{\mathfrak{MD}_2}\mathfrak{K}_2^{\mathfrak{MD}_2},\dots,\mathfrak{K}_1^{\mathfrak{MD}_m}\mathfrak{K}_2^{\mathfrak{MD}_m}\right)$$

$$\lambda\mathfrak{K}_1^{\mathcal{M}_p} = \left(1-\left(1-\mathfrak{K}_1^{\mathfrak{MD}_1}\right)^{\lambda},1-\left(1-\mathfrak{K}_1^{\mathfrak{MD}_2}\right)^{\lambda},\dots,1-\left(1-\mathfrak{K}_1^{\mathfrak{MD}_m}\right)^{\lambda}\right)$$

$$\left(\mathfrak{K}_1^{\mathcal{M}_p}\right)^{\lambda} = \left(\left(\mathfrak{K}_1^{\mathfrak{MD}_1}\right)^{\lambda},\left(\mathfrak{K}_1^{\mathfrak{MD}_2}\right)^{\lambda},\dots,\left(\mathfrak{K}_1^{\mathfrak{MD}_m}\right)^{\lambda}\right)$$

c. M-Polar Fuzzy Rough Linguistic Set

In this section, we discussed the concept of m-polar fuzzy rough linguistic set and derived its fundamental operational laws. We also discussed the novel operational laws of Frank aggregation operators under an m-polar fuzzy rough linguistic environment. Based on the proposed Frank operational laws, we developed Frank aggregation operators.

Definition 6: [21] Assume $\mathfrak{K}^{\mathfrak{U}_a}$ shows a universal set of discourse such that $\mathfrak{K}^{\mathfrak{U}_a} = (\mathfrak{K}^{u_1},\mathfrak{K}^{u_2},\mathfrak{K}^{u_3},\dots,\mathfrak{K}^{u_n})$. Then, the m-polar fuzzy rough linguistic set is defined as

$$\mathfrak{K}^{\mathcal{M}_\mathfrak{f}} = \left\{ \left(\mathfrak{K}^{u_n}, \left(\mathfrak{L}_{\mathfrak{K}^{\mathbb{l}_i}(\mathfrak{K}^{u_n})}, \left(\begin{array}{c} \left(\underline{\mathfrak{K}^{\mathfrak{MD}_1}}(\mathfrak{K}^{u_n}), \underline{\mathfrak{K}^{\mathfrak{MD}_2}}(\mathfrak{K}^{u_n}), \underline{\mathfrak{K}^{\mathfrak{MD}_3}}(\mathfrak{K}^{u_n}), \ldots, \underline{\mathfrak{K}^{\mathfrak{MD}_m}}(\mathfrak{K}^{u_n}) \right), \\ \left(\overline{\mathfrak{K}^{\mathfrak{MD}_1}}(\mathfrak{K}^{u_n}), \overline{\mathfrak{K}^{\mathfrak{MD}_2}}(\mathfrak{K}^{u_n}), \overline{\mathfrak{K}^{\mathfrak{MD}_3}}(\mathfrak{K}^{u_n}), \ldots, \overline{\mathfrak{K}^{\mathfrak{MD}_m}}(\mathfrak{K}^{u_n}) \right) \end{array} \right) \right) \right) \mathfrak{K}^{u_n} \in \mathfrak{K}^{\mathfrak{U}_a} \right\}$$

Where $\left(\underline{\mathfrak{K}^{\mathfrak{MD}_1}}(\mathfrak{K}^{u_n}), \underline{\mathfrak{K}^{\mathfrak{MD}_2}}(\mathfrak{K}^{u_n}), \underline{\mathfrak{K}^{\mathfrak{MD}_3}}(\mathfrak{K}^{u_n}), \ldots, \underline{\mathfrak{K}^{\mathfrak{MD}_m}}(\mathfrak{K}^{u_n}) \right)$ shows a family of lower approximation membership degrees and

$$\left(\overline{\mathfrak{K}^{\mathfrak{MD}_1}}(\mathfrak{K}^{u_n}), \overline{\mathfrak{K}^{\mathfrak{MD}_2}}(\mathfrak{K}^{u_n}), \overline{\mathfrak{K}^{\mathfrak{MD}_3}}(\mathfrak{K}^{u_n}), \ldots, \overline{\mathfrak{K}^{\mathfrak{MD}_m}}(\mathfrak{K}^{u_n}) \right)$$

shows a family of upper approximation membership degrees that fulfils the restrictions that each element of both lower and upper approximation membership families belongs to the closed interval of 0 and 1, and the term $\mathfrak{K}^{\mathbb{l}_i}(\mathfrak{K}^{u_n})$ represents a continuous linguistic number, having the condition that $\mathfrak{K}^{\mathbb{l}_i}(\mathfrak{K}^{u_n}) \in [0, \Upsilon]$.

An m-polar fuzzy rough linguistic number (m-PFRLN) is represented by

$$\mathfrak{K}_{\mathbb{N}}^{\mathcal{M}_\mathfrak{f}} = \left(\mathfrak{L}_{\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_i}}, \left(\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_1}}, \underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_2}}, \underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_3}}, \ldots, \underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_m}} \right), \left(\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_1}}, \overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_2}}, \overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_3}}, \ldots, \overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_m}} \right) \right).$$

Now, let us consider

$$\mathfrak{K}_1^{\mathcal{M}_\mathfrak{f}} = \left(\mathfrak{L}_{\mathfrak{K}_1^{\mathbb{l}_i}}, \left(\underline{\mathfrak{K}_1^{\mathfrak{MD}_1}}, \underline{\mathfrak{K}_1^{\mathfrak{MD}_2}}, \underline{\mathfrak{K}_1^{\mathfrak{MD}_3}}, \ldots, \underline{\mathfrak{K}_1^{\mathfrak{MD}_m}} \right), \left(\overline{\mathfrak{K}_1^{\mathfrak{MD}_1}}, \overline{\mathfrak{K}_1^{\mathfrak{MD}_2}}, \overline{\mathfrak{K}_1^{\mathfrak{MD}_3}}, \ldots, \overline{\mathfrak{K}_1^{\mathfrak{MD}_m}} \right) \right),$$

and $\mathfrak{K}_2^{\mathcal{M}_\mathfrak{f}} = \left(\mathfrak{L}_{\mathfrak{K}_2^{\mathbb{l}_i}}, \left(\underline{\mathfrak{K}_2^{\mathfrak{MD}_1}}, \underline{\mathfrak{K}_2^{\mathfrak{MD}_2}}, \underline{\mathfrak{K}_2^{\mathfrak{MD}_3}}, \ldots, \underline{\mathfrak{K}_2^{\mathfrak{MD}_m}} \right), \left(\overline{\mathfrak{K}_2^{\mathfrak{MD}_1}}, \overline{\mathfrak{K}_2^{\mathfrak{MD}_2}}, \overline{\mathfrak{K}_2^{\mathfrak{MD}_3}}, \ldots, \overline{\mathfrak{K}_2^{\mathfrak{MD}_m}} \right) \right)$

be two m-PFRLN. Then, the basic operational laws for all m-PFRLN are defined as

$$\mathfrak{K}_1^{\mathcal{M}_\mathfrak{f}} \oplus \mathfrak{K}_2^{\mathcal{M}_\mathfrak{f}} = \left(\begin{array}{c} \mathfrak{L}_{\Upsilon\left(\frac{\mathfrak{K}_1^{\mathbb{l}_i}}{\Upsilon} + \frac{\mathfrak{K}_2^{\mathbb{l}_i}}{\Upsilon} - \frac{\mathfrak{K}_1^{\mathbb{l}_i}}{\Upsilon}\frac{\mathfrak{K}_2^{\mathbb{l}_i}}{\Upsilon}\right)}, \\ \left(\begin{array}{c} \left(\underline{\mathfrak{K}_1^{\mathfrak{MD}_1}} + \underline{\mathfrak{K}_2^{\mathfrak{MD}_1}} - \underline{\mathfrak{K}_1^{\mathfrak{MD}_1}\mathfrak{K}_2^{\mathfrak{MD}_1}}, \underline{\mathfrak{K}_1^{\mathfrak{MD}_2}} + \underline{\mathfrak{K}_2^{\mathfrak{MD}_2}} - \underline{\mathfrak{K}_1^{\mathfrak{MD}_2}\mathfrak{K}_2^{\mathfrak{MD}_2}}, \ldots, \underline{\mathfrak{K}_1^{\mathfrak{MD}_m}} + \underline{\mathfrak{K}_2^{\mathfrak{MD}_m}} - \underline{\mathfrak{K}_1^{\mathfrak{MD}}} \right. \\ \left(\overline{\mathfrak{K}_1^{\mathfrak{MD}_1}} + \overline{\mathfrak{K}_2^{\mathfrak{MD}_1}} - \overline{\mathfrak{K}_1^{\mathfrak{MD}_1}\mathfrak{K}_2^{\mathfrak{MD}_1}}, \overline{\mathfrak{K}_1^{\mathfrak{MD}_2}} + \overline{\mathfrak{K}_2^{\mathfrak{MD}_2}} - \overline{\mathfrak{K}_1^{\mathfrak{MD}_2}\mathfrak{K}_2^{\mathfrak{MD}_2}}, \ldots, \overline{\mathfrak{K}_1^{\mathfrak{MD}_m}} + \overline{\mathfrak{K}_2^{\mathfrak{MD}_m}} - \overline{\mathfrak{K}_1^{\mathfrak{MT}}} \right. \end{array} \right. \end{array} \right.$$

$$\mathfrak{K}_1^{\mathcal{M}_\mathfrak{f}} \otimes \mathfrak{K}_2^{\mathcal{M}_\mathfrak{f}} = \left(\begin{array}{c} \mathfrak{L}_{\Upsilon\left(\frac{\mathfrak{K}_1^{\mathbb{l}_i}}{\Upsilon}\frac{\mathfrak{K}_2^{\mathbb{l}_i}}{\Upsilon}\right)}, \\ \left(\left(\underline{\mathfrak{K}_1^{\mathfrak{MD}_1}\mathfrak{K}_2^{\mathfrak{MD}_1}}, \underline{\mathfrak{K}_1^{\mathfrak{MD}_2}\mathfrak{K}_2^{\mathfrak{MD}_2}}, \ldots, \underline{\mathfrak{K}_1^{\mathfrak{MD}_m}\mathfrak{K}_2^{\mathfrak{MD}_m}} \right), \left(\overline{\mathfrak{K}_1^{\mathfrak{MD}_1}\mathfrak{K}_2^{\mathfrak{MD}_1}}, \overline{\mathfrak{K}_1^{\mathfrak{MD}_2}\mathfrak{K}_2^{\mathfrak{MD}_2}}, \ldots, \overline{\mathfrak{K}_1^{\mathfrak{MD}_m}\mathfrak{K}_2^{\mathfrak{MD}_m}} \right) \right) \end{array} \right)$$

$$\lambda\mathfrak{K}_1^{\mathcal{M}_\mathfrak{f}} = \left(\begin{array}{c} \mathfrak{L}_{\Upsilon\left(1 - \left(1 - \frac{\mathfrak{K}_1^{\mathbb{l}_i}}{\Upsilon}\right)^{\lambda}\right)}, \\ \left(\begin{array}{c} \left(1 - \left(1 - \underline{\mathfrak{K}_1^{\mathfrak{MD}_1}}\right)^{\lambda}, 1 - \left(1 - \underline{\mathfrak{K}_1^{\mathfrak{MD}_2}}\right)^{\lambda}, \ldots, 1 - \left(1 - \underline{\mathfrak{K}_1^{\mathfrak{MD}_m}}\right)^{\lambda} \right), \\ \left(1 - \left(1 - \overline{\mathfrak{K}_1^{\mathfrak{MD}_1}}\right)^{\lambda}, 1 - \left(1 - \overline{\mathfrak{K}_1^{\mathfrak{MD}_2}}\right)^{\lambda}, \ldots, 1 - \left(1 - \overline{\mathfrak{K}_1^{\mathfrak{MD}_m}}\right)^{\lambda} \right) \end{array} \right) \end{array} \right)$$

$$\left(\mathfrak{K}_1^{\mathcal{M}_\mathfrak{f}}\right)^{\lambda} = \left(\mathfrak{L}_{\Upsilon\left(\frac{\mathfrak{K}_1^{\mathbb{l}_i}}{\Upsilon}\right)^{\lambda}}, \left(\left(\left(\underline{\mathfrak{K}_1^{\mathfrak{MD}_1}}\right)^{\lambda}, \left(\underline{\mathfrak{K}_1^{\mathfrak{MD}_2}}\right)^{\lambda}, \ldots, \left(\underline{\mathfrak{K}_1^{\mathfrak{MD}_m}}\right)^{\lambda} \right), \left(\left(\overline{\mathfrak{K}_1^{\mathfrak{MD}_1}}\right)^{\lambda}, \left(\overline{\mathfrak{K}_1^{\mathfrak{MD}_2}}\right)^{\lambda}, \ldots, \left(\overline{\mathfrak{K}_1^{\mathfrak{MD}_m}}\right)^{\lambda} \right) \right) \right)$$

Definition 7: Assume that

$$\mathfrak{K}_{\mathbb{N}}^{\mathcal{M}_\mathfrak{f}} = \left(\mathfrak{L}_{\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_i}}, \left(\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_1}}, \underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_2}}, \underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_3}}, \ldots, \underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_m}} \right), \left(\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_1}}, \overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_2}}, \overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_3}}, \ldots, \overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_m}} \right) \right)$$

be an m-PFRLN. Then, for all m-PFRLN, the score function is defined as

$$\mathfrak{K}^{\mathfrak{S}^{\mathfrak{f}}}\left(\mathfrak{K}_{\mathbb{N}}^{\mathcal{M}_\mathfrak{f}}\right) = \frac{1}{3}\left(\left(\frac{\mathfrak{L}_{\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_i}}}{\Upsilon} \right) + \frac{1}{m}\sum_{i=1}^{m} \underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_i}} + \frac{1}{m}\sum_{i=1}^{m} \overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_i}} \right)$$

Definition 8: Let us consider

$$\mathfrak{K}_1^{\mathcal{M}_\mathfrak{f}} = \left(\mathfrak{L}_{\mathfrak{K}_1^{\mathbb{l}_i}}, \left(\underline{\mathfrak{K}_1^{\mathfrak{MD}_1}}, \underline{\mathfrak{K}_1^{\mathfrak{MD}_2}}, \underline{\mathfrak{K}_1^{\mathfrak{MD}_3}}, \ldots, \underline{\mathfrak{K}_1^{\mathfrak{MD}_m}} \right), \left(\overline{\mathfrak{K}_1^{\mathfrak{MD}_1}}, \overline{\mathfrak{K}_1^{\mathfrak{MD}_2}}, \overline{\mathfrak{K}_1^{\mathfrak{MD}_3}}, \ldots, \overline{\mathfrak{K}_1^{\mathfrak{MD}_m}} \right) \right),$$

And

$$\mathfrak{K}_2^{\mathcal{M}_\mathfrak{f}} = \left(\mathfrak{L}_{\mathfrak{K}_2^{\mathbb{l}_i}}, \left(\underline{\mathfrak{K}_2^{\mathfrak{MD}_1}}, \underline{\mathfrak{K}_2^{\mathfrak{MD}_2}}, \underline{\mathfrak{K}_2^{\mathfrak{MD}_3}}, \ldots, \underline{\mathfrak{K}_2^{\mathfrak{MD}_m}} \right), \left(\overline{\mathfrak{K}_2^{\mathfrak{MD}_1}}, \overline{\mathfrak{K}_2^{\mathfrak{MD}_2}}, \overline{\mathfrak{K}_2^{\mathfrak{MD}_3}}, \ldots, \overline{\mathfrak{K}_2^{\mathfrak{MD}_m}} \right) \right)$$

be two m-PFRLN. Then, for all m-PFRLN, the Frank operational laws are developed as

$$\mathfrak{K}_1^{\mathcal{M}_\mathfrak{f}} \oplus \mathfrak{K}_2^{\mathcal{M}_\mathfrak{f}} = \left(\begin{array}{c} \mathfrak{L}_{\Upsilon\left(1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\frac{\mathfrak{K}_1^{\mathbb{l}_i}}{\Upsilon}} - 1\right)\left(\mathfrak{I}^{1-\frac{\mathfrak{K}_2^{\mathbb{l}_i}}{\Upsilon}} - 1\right)}{\mathfrak{I} - 1}\right)\right)}, \\ \left(\begin{array}{c} \left(\begin{array}{c} 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_1^{\mathfrak{MD}_1}}} - 1\right)\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_2^{\mathfrak{MD}_1}}} - 1\right)}{\mathfrak{I} - 1}\right), 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_1^{\mathfrak{MD}_2}}} - 1\right)\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_2^{\mathfrak{MD}_2}}} - 1\right)}{\mathfrak{I} - 1}\right), \ldots, \\ 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_1^{\mathfrak{MD}_m}}} - 1\right)\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_2^{\mathfrak{MD}_m}}} - 1\right)}{\mathfrak{I} - 1}\right) \end{array} \right), \\ \left(\begin{array}{c} 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_1^{\mathfrak{MD}_1}}} - 1\right)\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_2^{\mathfrak{MD}_1}}} - 1\right)}{\mathfrak{I} - 1}\right), 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_1^{\mathfrak{MD}_2}}} - 1\right)\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_2^{\mathfrak{MD}_2}}} - 1\right)}{\mathfrak{I} - 1}\right), \ldots, \\ 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_1^{\mathfrak{MD}_m}}} - 1\right)\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_2^{\mathfrak{MD}_m}}} - 1\right)}{\mathfrak{I} - 1}\right) \end{array} \right) \end{array} \right) \end{array} \right)$$

$$\mathfrak{K}_1^{\mathcal{M}_\mathfrak{f}} \oplus \mathfrak{K}_2^{\mathcal{M}_\mathfrak{f}} = \left(\begin{array}{c} \mathfrak{L}_{\Upsilon\left(1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\frac{\mathfrak{K}_1^{\mathbb{l}_i}}{\Upsilon}} - 1\right)\left(\mathfrak{I}^{1-\frac{\mathfrak{K}_2^{\mathbb{l}_i}}{\Upsilon}} - 1\right)}{\mathfrak{I} - 1}\right)\right)}, \\ \left(\begin{array}{c} \left(\begin{array}{c} 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_1^{\mathfrak{MD}_1}}} - 1\right)\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_2^{\mathfrak{MD}_1}}} - 1\right)}{\mathfrak{I} - 1}\right), 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_1^{\mathfrak{MD}_2}}} - 1\right)\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_2^{\mathfrak{MD}_2}}} - 1\right)}{\mathfrak{I} - 1}\right), \ldots, \\ 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_1^{\mathfrak{MD}_m}}} - 1\right)\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_2^{\mathfrak{MD}_m}}} - 1\right)}{\mathfrak{I} - 1}\right) \end{array} \right), \\ \left(\begin{array}{c} 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_1^{\mathfrak{MD}_1}}} - 1\right)\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_2^{\mathfrak{MD}_1}}} - 1\right)}{\mathfrak{I} - 1}\right), 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_1^{\mathfrak{MD}_2}}} - 1\right)\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_2^{\mathfrak{MD}_2}}} - 1\right)}{\mathfrak{I} - 1}\right), \ldots, \\ 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_1^{\mathfrak{MD}_m}}} - 1\right)\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_2^{\mathfrak{MD}_m}}} - 1\right)}{\mathfrak{I} - 1}\right) \end{array} \right) \end{array} \right) \end{array} \right)$$

$$= \left(\begin{array}{c} \mathfrak{L}_{\Upsilon\left(1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\frac{\mathfrak{K}_1^{\mathbb{l}_i}}{\Upsilon}} - 1\right)^{\lambda}}{(\mathfrak{I} - 1)^{\lambda-1}}\right)\right)}, \\ \left(\begin{array}{c} \left(1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_1^{\mathfrak{MD}_1}}} - 1\right)^{\lambda}}{(\mathfrak{I} - 1)^{\lambda-1}}\right), 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_1^{\mathfrak{MD}_2}}} - 1\right)^{\lambda}}{(\mathfrak{I} - 1)^{\lambda-1}}\right), \ldots, 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_1^{\mathfrak{MD}_m}}} - 1\right)^{\lambda}}{(\mathfrak{I} - 1)^{\lambda-1}}\right) \right), \\ \left(1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_1^{\mathfrak{MD}_1}}} - 1\right)^{\lambda}}{(\mathfrak{I} - 1)^{\lambda-1}}\right), 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_1^{\mathfrak{MD}_2}}} - 1\right)^{\lambda}}{(\mathfrak{I} - 1)^{\lambda-1}}\right), \ldots, 1 - \log_{\mathfrak{I}}\left(1 + \frac{\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_1^{\mathfrak{MD}_m}}} - 1\right)^{\lambda}}{(\mathfrak{I} - 1)^{\lambda-1}}\right) \right) \end{array} \right) \end{array} \right)$$

$$\left(\mathfrak{K}_1^{\mathcal{M}_{\mathfrak{f}}}\right)^{\lambda}=\left(\begin{array}{c}\mathfrak{L}_{\Upsilon\left(\log_{\mathfrak{I}}\left(1+\frac{\left(\mathfrak{I}^{\frac{\mathfrak{K}_1^{\mathbb{l}_i}}{\Upsilon}}-1\right)^{\lambda}}{(\mathfrak{I}-1)^{\lambda-1}}\right)\right)},\\ \left(\begin{array}{c}\left(\log_{\mathfrak{I}}\left(1+\frac{\left(\mathfrak{I}^{\underline{\mathfrak{K}_1^{\mathfrak{MD}_1}}}-1\right)^{\lambda}}{(\mathfrak{I}-1)^{\lambda-1}}\right),\log_{\mathfrak{I}}\left(1+\frac{\left(\mathfrak{I}^{\underline{\mathfrak{K}_1^{\mathfrak{MD}_2}}}-1\right)^{\lambda}}{(\mathfrak{I}-1)^{\lambda-1}}\right),\ldots,\log_{\mathfrak{I}}\left(1+\frac{\left(\mathfrak{I}^{\underline{\mathfrak{K}_1^{\mathfrak{MD}_m}}}-1\right)^{\lambda}}{(\mathfrak{I}-1)^{\lambda-1}}\right)\right),\\ \left(\log_{\mathfrak{I}}\left(1+\frac{\left(\mathfrak{I}^{\overline{\mathfrak{K}_1^{\mathfrak{MD}_1}}}-1\right)^{\lambda}}{(\mathfrak{I}-1)^{\lambda-1}}\right),\log_{\mathfrak{I}}\left(1+\frac{\left(\mathfrak{I}^{\overline{\mathfrak{K}_1^{\mathfrak{MD}_2}}}-1\right)^{\lambda}}{(\mathfrak{I}-1)^{\lambda-1}}\right),\ldots,\log_{\mathfrak{I}}\left(1+\frac{\left(\mathfrak{I}^{\overline{\mathfrak{K}_1^{\mathfrak{MD}_m}}}-1\right)^{\lambda}}{(\mathfrak{I}-1)^{\lambda-1}}\right)\right)\end{array}\right)\end{array}\right)$$

Definition 9: Let us suppose that $\mathfrak{K}_{\mathbb{N}}^{\mathcal{M}_{\mathfrak{f}_i}}=\left(\mathfrak{L}_{\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}}},\left(\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}},\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}},\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{3_i}}},\ldots,\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}\right),\left(\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}},\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}},\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{3_i}}},\ldots,\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}\right)\right)$ be

an m-PFRLN such that $\left(\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}},\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}},\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{3_i}}},\ldots,\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}\right)$ represents a family of lower approximation membership degrees while

$$\left(\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}},\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}},\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{3_i}}},\ldots,\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}\right)$$

represents a family of upper approximation membership degrees having the conditions that each element of both families belongs to the closed interval of 0 and 1.

The term $\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}}$ represents a continuous linguistic number such that $\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}}\in[0,\Upsilon]$, and a weight vector is shown by $\mathfrak{K}^{\mathbb{W}_i}=(\mathfrak{K}^{\mathbb{W}_1},\mathfrak{K}^{\mathbb{W}_2},\mathfrak{K}^{\mathbb{W}_3},\ldots,\mathfrak{K}^{\mathbb{W}_n})$,
Having the condition that $\sum_{i=1}^{n}\mathfrak{K}^{\mathbb{W}_i}=1$.
Then, an m-polar fuzzy rough linguistic Frank weighted averaging (m-PFRLFWA) aggregation operator is defined as

$$m-PFRLFWA\left(\mathfrak{K}_1^{\mathcal{M}_{\mathfrak{f}_i}},\mathfrak{K}_2^{\mathcal{M}_{\mathfrak{f}_i}},\mathfrak{K}_3^{\mathcal{M}_{\mathfrak{f}_i}},\ldots,\mathfrak{K}_{\mathbb{N}}^{\mathcal{M}_{\mathfrak{f}_i}}\right)=\oplus_{i=1}^{n}\left(\mathfrak{K}^{\mathbb{W}_i}\mathfrak{K}_{\mathbb{N}}^{\mathcal{M}_{\mathfrak{f}_i}}\right)$$

$$=\left(\begin{array}{c}\mathfrak{L}_{\Upsilon\left(1-\log_{\mathfrak{I}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{I}^{1-\left(\frac{\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}}}{\Upsilon}\right)}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right)\right)},\\ \left(\begin{array}{c}\left(\begin{array}{c}1-\log_{\mathfrak{I}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right),1-\log_{\mathfrak{I}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right),\ldots,\\ 1-\log_{\mathfrak{I}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{I}^{1-\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right)\end{array}\right),\\ \left(\begin{array}{c}1-\log_{\mathfrak{I}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right),1-\log_{\mathfrak{I}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right),\ldots,\\ 1-\log_{\mathfrak{I}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{I}^{1-\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right)\end{array}\right)\end{array}\right)\end{array}\right)$$

Definition 10:

Let us suppose that

$$\mathfrak{K}_{\mathbb{N}}^{\mathcal{M}_{\mathfrak{f}_i}}=\left(\mathfrak{L}_{\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}}},\left(\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}},\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}},\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{3_i}}},\ldots,\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}\right),\left(\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}},\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}},\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{3_i}}},\ldots,\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}\right)\right)$$

be an m-PFRLN such that

$$\left(\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}},\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}},\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{3_i}}},\ldots,\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}\right)$$

represents a family of lower approximation membership degrees while

$$\left(\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}},\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}},\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{3_i}}},\ldots,\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}\right)$$

represents a family of upper approximation membership degrees having the conditions that each element of both families belongs to the closed interval of 0 and 1.

The term $\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}}$ represents a continuous linguistic number such that $\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}}\in[0,\Upsilon]$,
and a weight vector is shown by $\mathfrak{K}^{\mathbb{W}_i}=(\mathfrak{K}^{\mathbb{W}_1},\mathfrak{K}^{\mathbb{W}_2},\mathfrak{K}^{\mathbb{W}_3},\ldots,\mathfrak{K}^{\mathbb{W}_n})$, having the condition that $\sum_{i=1}^{n}\mathfrak{K}^{\mathbb{W}_i}=1$. Then, an m-polar fuzzy rough linguistic Frank weighted geometric (m-PFRLFWG) aggregation operator is defined as

$$m-PFRLFWG\left(\mathfrak{K}_1^{\mathcal{M}_{\mathfrak{f}_i}},\mathfrak{K}_2^{\mathcal{M}_{\mathfrak{f}_i}},\mathfrak{K}_3^{\mathcal{M}_{\mathfrak{f}_i}},\ldots,\mathfrak{K}_{\mathbb{N}}^{\mathcal{M}_{\mathfrak{f}_i}}\right)=\otimes_{i=1}^{n}\left(\mathfrak{K}_{\mathbb{N}}^{\mathcal{M}_{\mathfrak{f}_i}}\right)^{\mathfrak{K}^{\mathbb{W}_i}}$$

$$=\left(\begin{array}{c}\mathfrak{L}_{\Upsilon\left(\log_{\mathfrak{I}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{I}^{\left(\frac{\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}}}{\Upsilon}\right)}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right)\right)},\\ \left(\begin{array}{c}\left(\begin{array}{c}\log_{\mathfrak{I}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{I}^{\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right),\log_{\mathfrak{I}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{I}^{\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right),\ldots,\\ \log_{\mathfrak{I}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{I}^{\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right)\end{array}\right),\\ \left(\begin{array}{c}\log_{\mathfrak{I}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{I}^{\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right),\log_{\mathfrak{I}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{I}^{\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right),\ldots,\\ \log_{\mathfrak{I}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{I}^{\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right)\end{array}\right)\end{array}\right)\end{array}\right)$$

d. WASPAS Model Based on M-Polar Fuzzy Rough Linguistic Information

The WASPAS model is a well-known MADM technique, widely used across different domains to evaluate and rank alternatives based on multiple criteria. The WASPAS approach is widely known for its flexibility, simplicity, and strong decision accuracy. The WASPAS approach integrates the strengths of two different decision-making models, such as the Weighted Sum Model and Weighted Product Model. This combination enables the WASPAS approach to provide a more accurate and balanced final ranking result of the alternatives. By means of this combination, WASPAS also reduces bias in the operational process, which may arise when depending on a single approach. This approach is highly effective in complex decision environments where alternatives are evaluated under multiple conflicting criteria. It enables the decision makers to assign weights to each attribute according to their importance effectively, which makes the WASPAS approach highly adaptable to different domains, including engineering, information management, computer science, sustainability, and supply chain management. It also effective addresses both types of expert information, such as benefits or costs, which increases models' usefulness. The WASPAS approach easily integrates with different uncertainty handling approaches, such as fuzzy, linguistic, or any other hybrid approach. This phenomenon enables the WASPAS model to handle real-world decision-making problems, especially when the information is expressed in qualitative terms instead of precise numerical values. Another key advantage of the WASPAS model is that it gives a clear and accurate final ranking result of the alternatives, which enables the decision makers to easily get the most accurate alternative from the selected set of alternatives. The operational process of the WASPAS approach is very clear and transparent, and that is why it can be easily implemented and understands for each user. It allows the decision makers to clearly understand the operational process and know the effect of each attribute on the final result.

In this research work, we extended the existing notation of the WASPAS model to an m-polar fuzzy rough linguistic environment. This extension enhances the evaluation and ranking approach of the WASPAS model. It makes the WASPAS approach more reliable for complex and uncertain real-world decision-making problems. Using this generalized approach; decision makers can easily handle complex and uncertain expert information. The working mechanism of the generalized WASPAS model is very simple, and everyone can easily understand and implement it.

To explain each step of the WASPAS model, first, we assume that $(\mathfrak{K}^{\mathfrak{a}_{\mathfrak{t}(1)}}, \mathfrak{K}^{\mathfrak{a}_{\mathfrak{t}(2)}}, \mathfrak{K}^{\mathfrak{a}_{\mathfrak{t}(3)}}, \dots, \mathfrak{K}^{\mathfrak{a}_{\mathfrak{t}(n)}})$a set of alternatives and $(\mathfrak{K}^{\mathfrak{a}_{\mathfrak{t}(1)}\prime}, \mathfrak{K}^{\mathfrak{a}_{\mathfrak{t}(2)}\prime}, \mathfrak{K}^{\mathfrak{a}_{\mathfrak{t}(3)}\prime}, \dots, \mathfrak{K}^{\mathfrak{a}_{\mathfrak{t}(n)}\prime})$ be a set of attributes. Now, the complete operational process of the extended WASPAS approach is discussed below:

Step 1: Generate a decision matrix from the expert judgment expressed in m-polar fuzzy rough linguistic form. The decision matrix will be in the following shape.

$$\mathfrak{K}_{\mathbb{D}^m} = [\mathfrak{y}_{ij}]_{m\times n} = \begin{bmatrix} \mathfrak{y}_{11} & \mathfrak{y}_{12} & \cdots & \mathfrak{y}_{1n} \\ \mathfrak{y}_{21} & \mathfrak{y}_{22} & \cdots & \mathfrak{y}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathfrak{y}_{m2} & \mathfrak{y}_{m2} & \cdots & \mathfrak{y}_{mn} \end{bmatrix}$$

Step 2: Construct a normalized decision matrix from the information located in *Step 1*. The following formula is used for constructing the normalized decision matrix.

$$\mathfrak{K}_{\mathcal{N}\mathbb{D}^m} = \left(\begin{array}{c} \left(\begin{array}{c} \mathfrak{L}_{\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}} = \frac{\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}}}{1+\max\left(\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}}\right)}}, \\ \left(\begin{array}{c} \left(\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}} = \frac{\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}}{1+\max\left(\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}\right)}, \underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}} = \frac{\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}}{1+\max\left(\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}\right)}, \dots, \underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}} = \frac{\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}}{1+\max\left(\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}\right)} \right), \\ \left(\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}} = \frac{\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}}{1+\max\left(\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}\right)}, \overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}} = \frac{\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}}{1+\max\left(\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}\right)}, \dots, \overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}} = \frac{\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}}{1+\max\left(\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}\right)} \right) \end{array} \right) \end{array} \right), \\ \left(\begin{array}{c} \mathfrak{L}_{\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}} = \frac{\min\left(\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}}\right)}{1+\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}}}}, \\ \left(\begin{array}{c} \left(\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}} = \frac{\min\left(\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}\right)}{1+\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}}, \underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}} = \frac{\min\left(\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}\right)}{1+\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}}, \dots, \underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}} = \frac{\min\left(\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}\right)}{1+\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}} \right), \\ \left(\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}} = \frac{\min\left(\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}\right)}{1+\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}}, \overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}} = \frac{\min\left(\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}\right)}{1+\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}}, \dots, \overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}} = \frac{\min\left(\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}\right)}{1+\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}} \right) \end{array} \right) \end{array} \right) \end{array} \right)$$

Step 3: Determine the weighted sum and weighted product model from the data located in *Step 2*. The following formulas are used for calculating both the weighted sum and product models.

$$\mathfrak{K}^{\mathbb{Q}_1^i} = m - PFRLFWA\left(\mathfrak{K}_1^{\mathcal{M}_{\mathfrak{f}_i}}, \mathfrak{K}_2^{\mathcal{M}_{\mathfrak{f}_i}}, \mathfrak{K}_3^{\mathcal{M}_{\mathfrak{f}_i}}, \dots, \mathfrak{K}_{\mathbb{N}}^{\mathcal{M}_{\mathfrak{f}_i}}\right)$$

$$= \left(\begin{array}{c} \mathfrak{L}_{\Upsilon\left(1-\log_{\mathfrak{J}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{J}^{1-\left(\frac{\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}}}{\Upsilon}\right)}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right)\right)}, \\ \left(\begin{array}{c} \left(\begin{array}{c} 1-\log_{\mathfrak{J}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{J}^{1-\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right), 1-\log_{\mathfrak{J}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{J}^{1-\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right), \dots, \\ 1-\log_{\mathfrak{J}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{J}^{1-\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right) \end{array} \right), \\ \left(\begin{array}{c} 1-\log_{\mathfrak{J}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{J}^{1-\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right), 1-\log_{\mathfrak{J}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{J}^{1-\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right), \dots, \\ 1-\log_{\mathfrak{J}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{J}^{1-\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right) \end{array} \right) \end{array} \right) \end{array} \right)$$

$$\mathfrak{K}^{\mathbb{Q}_2^i} = m - PFRLFWG\left(\mathfrak{K}_1^{\mathcal{M}_{\mathfrak{f}_i}}, \mathfrak{K}_2^{\mathcal{M}_{\mathfrak{f}_i}}, \mathfrak{K}_3^{\mathcal{M}_{\mathfrak{f}_i}}, \dots, \mathfrak{K}_{\mathbb{N}}^{\mathcal{M}_{\mathfrak{f}_i}}\right)$$

$$= \left(\begin{array}{c} \mathfrak{L}_{\Upsilon\left(\log_{\mathfrak{J}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{J}^{\left(\frac{\mathfrak{K}_{\mathbb{N}}^{\mathbb{l}_{i_i}}}{\Upsilon}\right)}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right)\right)}, \\ \left(\begin{array}{c} \left(\begin{array}{c} \log_{\mathfrak{J}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{J}^{\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right), \log_{\mathfrak{J}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{J}^{\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right), \dots, \\ \log_{\mathfrak{J}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{J}^{\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right) \end{array} \right), \\ \left(\begin{array}{c} \log_{\mathfrak{J}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{J}^{\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right), \log_{\mathfrak{J}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{J}^{\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right), \dots, \\ \log_{\mathfrak{J}}\left(1+\prod_{i=1}^{n}\left(\mathfrak{J}^{\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{m_i}}}}-1\right)^{\mathfrak{K}^{\mathbb{W}_i}}\right) \end{array} \right) \end{array} \right) \end{array} \right)$$

Step 4: Calculate the final WASPAS aggregated score of each alternative using the information located in *Step 3*. The following formula is used for calculating the score of the alternatives.

$$\mathfrak{K}^{\mathbb{Q}_{\prime}^{i}} = \lambda * \mathfrak{K}^{\mathfrak{S}^{\mathfrak{f}}}\left(\mathfrak{K}^{\mathbb{Q}_{1}^{i}}\right) + (1-\lambda) * \mathfrak{K}^{\mathfrak{S}^{\mathfrak{f}}}\left(\mathfrak{K}^{\mathbb{Q}_{2}^{i}}\right)$$

Step 5: Rank the alternatives according to the final WASPAS score aggregated in *Step 4*. This ranking result helps the decision makers to select the best alternative from the given choices easily.

e. Selection of AI-Based Early Warning Systems for Natural Disaster Management Using WASPAS Model

Natural disasters, including landslides, floods, wildfires, cyclones, and earthquakes, are widely affecting human life, causing huge property damage and widespread economic disruption across the globe. These events occur very quickly, which leaves the communities unprepared to respond effectively. The impact of these disasters depends on different factors such as climatic conditions, geographical location, and population density. So, understanding of these factors is very effective for developing a reliable and efficient detection system. Early warning systems play an important role in natural disaster management and risk reduction. These systems enable the concerned authorities to take necessary prevention steps by providing timely alerts of a disaster. Natural diseases are of different type and each of them has a different cause and impact. For example, floods completely damage the infrastructure and mostly occur in South Asia and Southeast Asia. Earthquakes cause a huge environmental loss and mostly occur in regions like Australia, Japan, Taiwan, and California. So, this diversity clearly shows the effectiveness of an early warning system. Some natural disasters and their impact are briefly discussed in Table 3.

Table 3: Key natural disasters and their impact.

Type	*Impact*	*Regions*
Earthquakes	This disaster leads to severe casualties and environmental loss.	It mostly occurs in regions like California, Taiwan, Turkey, Japan, and Indonesia.
Landslides	It completely damages the property and causes road blockage, which may result in loss of human life and the economy.	It mostly occurs in mountainous regions across the world.
Floods	It results in loss of life, damages infrastructures, and also causes displacement.	It mostly occurs in the USA, Southeast Asia, and South Asia.
Wildfires	It affects the quality of the air, which disturbs the normal life of all living things and also causes habitat loss.	Widely occur in areas like the Mediterranean, the USA, and Australia.
Cyclones	It damages wind and causes flooding by means of heavy rain or coastal storms, damaging property and infrastructure.	Mostly occurs in India, the Philippines, and the Caribbean.

Furthermore, the integration of different AI technologies with these early warning systems enhances the efficiency and reliability of these systems and makes them highly effective for complex situations. These AI approaches enable these systems to effectively process large datasets and provide an accurate prediction. The effectiveness of all AI-based early warning systems completely depends on the quality and variety of the data. These AI approaches effectively handle diverse types of data source such as image data taken from the Satellite gives detailed information on weather patterns and land conditions. The environmental sensors placed on the ground provide information about rainfall and soil moisture. The AI-based early warning systems first analyze the data and learn the hidden patterns from it by means of advanced AI tools and then make prediction based on the learned information. Table 4 briefly presents the typical data sources for AI-based early warning systems.

Table 4: Some basic data sources for AI-based early warning systems.

Sources	*Explination*	*Role*
Environmental Sensors	All the ground-based devices effectively measure different parameters such as soil moisture, rainfall, etc.	This approach provides real-time local environmental data to early warning systems.
Historical Disaster Data	It contains all the previous records of disaster events and impacts.	It enables AI approaches to give an accurate prediction of the coming disasters earlier.

Satellite Imagery	It contains the information of earth surface in image form captured through a satellite.	It helps to monitor weather patterns, land changes, and vegetation.
Social media Feeds	It is open-access information and is publicly available from social platforms. Access to this information is very easy.	It helps to provide early detection of disaster reports and alerts.
Meteorological Reports	It contains weather forecasts and atmospheric data.	It enables the early warning systems to predict storms, weather events, and cyclones.

The complexity and diversity of natural disasters make it crucial for the decision makers to effectively evaluate such complex systems and get the best from them using traditional evaluation approaches. To overcome these challenges, a well-organized and reliable evaluation framework is required, which enables the decision makers to effectively handle such complex evaluation processes. Now, first we select the top five effective AI-based early warning systems that are widely used across the word and we take them as alternatives.

Each alternative is briefly explained below:

1. ***Wildfire Detection and Prediction Systems:*** These systems effectively monitor and forecast wildfires before they cause huge damage by means of advanced technologies. It integrates data from different sources such as weather stations, satellite images, and environmental sensors. The data collected from these devices is effectively analyzed by the AI algorithms and provides early signs. The key advantage of this approach is that it helps authorities to prevent large-scale damage and strengthens their response.

2. ***Cyclone Prediction Systems:*** These systems provide early information about the movement and intensity of cyclones. AI-based cyclone prediction systems effectively handle a large volume of data collected from different sources such as pecan buoys, satellites, and weather radars. The key advantage of this system is that it effectively identifies complex atmospheric patterns, which helps to determine the development of cyclones. It helps authorities to issue emergency plans, warnings, and take the necessary steps promptly. This approach helps to minimize human and economic losses caused by severe storms.

3. ***Landslide Monitoring Systems***: These systems are widely used in hilly and mountainous places to detect landslides earlier. The AI-based landslide monitoring systems effectively analyze the data integrated from sources like rainfall gauges, satellite imagery, and ground sensors. The sensors accurately measure multiple factors such as slope stability, ground movement, and soil moisture. These factors help to identify early signs of landslides. The key advantage of these systems is that it operates continuously and gives an accurate risk assessment.

4. ***Earthquake Early Warning System:*** These systems help to identify earthquakes and give early signs before the ground shakes. These approaches continuously monitor ground vibrations using specific sensors. AI-based earthquake early warning systems effectively analyze the information coming from sensors and identify earthquakes. The key advantage of these approaches is that its complete information of earthquake, including magnitude, location, etc. It helps people to take immediate protections including the power of the machinery, moving to a safe zone, and stopping traveling.

5. ***Flood Early Warning Systems:*** These systems are particularly designed to predict flooding events and give early alerts. An AI-based flood early warning system effectively analyzes data from weather forecasts, satellite imagery, and rainfall sensors. From this information, AI models detect water flow, flood risk, and possible areas of occurrence. The key advantage of these systems is that it effectively identifies expected flood timing and high-risk areas. These systems are known as one of the most reliable and effective warning systems.

The above alternatives show different early warning systems. To evaluate and rank these alternatives, we have to select a set of attributes. The set of selected attributes shows the effectiveness of each alternative.

The following is the selected set of alternatives.

1) Implementation and Operational Cost
2) Data Integration Capability
3) Prediction Accuracy
4) Response Time
5) Reliability and Robustness

Now, we will apply the proposed WASPAS model based on m-polar fuzzy rough linguistic information to evaluate and rank the above-mentioned alternatives. The fundamental steps of the proposed model are briefly explained below.

Step 1: Construct a decision matrix from the expert information expressed in m-polar fuzzy rough linguistic form. Table 5 presents the detailed expert information expressed in m-polar fuzzy rough linguistic from.

Table 5: Initial expert decision matrix.

$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(1)}}$	$\mathfrak{L}_{\mathfrak{K}_{\mathbb{N}}}^{\mathbb{l}_{i_i}}$	$\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}$	$\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}$	$\underline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{3_i}}}$	$\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}$	$\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}$	$\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{3_i}}}$
$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(1)}}{}'$	$\mathfrak{L}_1$	0.5	0.1	0.4	0.6	0.3	0.7
$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(2)}}{}'$	$\mathfrak{L}_2$	0.51	0.11	0.41	0.61	0.31	0.71
$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(3)}}{}'$	$\mathfrak{L}_4$	0.52	0.12	0.42	0.62	0.32	0.72
$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(4)}}{}'$	$\mathfrak{L}_6$	0.53	0.13	0.43	0.63	0.33	0.73
$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(5)}}{}'$	$\mathfrak{L}_7$	0.54	0.14	0.44	0.64	0.34	0.74
$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(2)}}$							
$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(1)}}{}'$	$\mathfrak{L}_2$	0.3	0.5	0.2	0.4	0.7	0.3
$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(2)}}{}'$	$\mathfrak{L}_3$	0.31	0.51	0.21	0.41	0.71	0.31
$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(3)}}{}'$	$\mathfrak{L}_5$	0.32	0.52	0.22	0.42	0.72	0.32
$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(4)}}{}'$	$\mathfrak{L}_7$	0.33	0.53	0.23	0.43	0.73	0.33
$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(5)}}{}'$	$\mathfrak{L}_8$	0.34	0.54	0.24	0.44	0.74	0.34

$\mathfrak{K}^{\mathfrak{a}_{t(3)}}$							
$\mathfrak{K}^{\mathfrak{a}_{t(1)}\prime}$	$\mathfrak{L}_1$	0.3	0.6	0.1	0.5	0.7	0.4
$\mathfrak{K}^{\mathfrak{a}_{t(2)}\prime}$	$\mathfrak{L}_3$	0.31	0.61	0.11	0.51	0.71	0.41
$\mathfrak{K}^{\mathfrak{a}_{t(3)}\prime}$	$\mathfrak{L}_5$	0.32	0.62	0.12	0.52	0.72	0.42
$\mathfrak{K}^{\mathfrak{a}_{t(4)}\prime}$	$\mathfrak{L}_6$	0.33	0.63	0.13	0.53	0.73	0.43
$\mathfrak{K}^{\mathfrak{a}_{t(5)}\prime}$	$\mathfrak{L}_8$	0.34	0.64	0.14	0.54	0.74	0.44
$\mathfrak{K}^{\mathfrak{a}_{t(4)}}$							
$\mathfrak{K}^{\mathfrak{a}_{t(1)}\prime}$	$\mathfrak{L}_3$	0.2	0.4	0.3	0.5	0.6	0.4
$\mathfrak{K}^{\mathfrak{a}_{t(2)}\prime}$	$\mathfrak{L}_4$	0.21	0.41	0.31	0.51	0.61	0.41
$\mathfrak{K}^{\mathfrak{a}_{t(3)}\prime}$	$\mathfrak{L}_5$	0.22	0.42	0.32	0.52	0.62	0.42
$\mathfrak{K}^{\mathfrak{a}_{t(4)}\prime}$	$\mathfrak{L}_7$	0.23	0.43	0.33	0.53	0.63	0.43
$\mathfrak{K}^{\mathfrak{a}_{t(5)}\prime}$	$\mathfrak{L}_8$	0.24	0.44	0.34	0.54	0.64	0.44
$\mathfrak{K}^{\mathfrak{a}_{t(5)}}$							
$\mathfrak{K}^{\mathfrak{a}_{t(1)}\prime}$	$\mathfrak{L}_2$	0.1	0.6	0.5	0.8	0.7	0.6
$\mathfrak{K}^{\mathfrak{a}_{t(2)}\prime}$	$\mathfrak{L}_3$	0.11	0.61	0.51	0.81	0.71	0.61
$\mathfrak{K}^{\mathfrak{a}_{t(3)}\prime}$	$\mathfrak{L}_4$	0.12	0.62	0.52	0.82	0.72	0.62
$\mathfrak{K}^{\mathfrak{a}_{t(4)}\prime}$	$\mathfrak{L}_6$	0.13	0.63	0.53	0.83	0.73	0.63
$\mathfrak{K}^{\mathfrak{a}_{t(5)}\prime}$	$\mathfrak{L}_8$	0.14	0.64	0.54	0.84	0.74	0.64

Step 2: Normalize the expert preference expressed in m-polar fuzzy rough linguistic form in Table 5. Table 6 presents the normalized decision matrix aggregated from the expert information.

Table 6: The normalized decision matrix.

$\mathfrak{K}^{\mathfrak{a}_{t(1)}}$	$\mathfrak{L}_{1_{i_{\mathfrak{K}_{\mathbb{N}}}}}$	$\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}$	$\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}$	$\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{3_i}}$	$\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{1_i}}}$	$\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{2_i}}}$	$\overline{\mathfrak{K}_{\mathbb{N}}^{\mathfrak{MD}_{3_i}}}$
$\mathfrak{K}^{\mathfrak{a}_{t(1)}\prime}$	$\mathfrak{L}_{0.125}$	0.3247	0.0877	0.2778	0.3659	0.2239	0.4023
$\mathfrak{K}^{\mathfrak{a}_{t(2)}\prime}$	$\mathfrak{L}_{0.25}$	0.3312	0.0965	0.2847	0.3720	0.2313	0.4081
$\mathfrak{K}^{\mathfrak{a}_{t(3)}\prime}$	$\mathfrak{L}_{0.5}$	0.3377	0.1053	0.2917	0.3781	0.2388	0.4138
$\mathfrak{K}^{\mathfrak{a}_{t(4)}\prime}$	$\mathfrak{L}_{0.75}$	0.3442	0.1140	0.2986	0.3842	0.2463	0.4195
$\mathfrak{K}^{\mathfrak{a}_{t(5)}\prime}$	$\mathfrak{L}_{0.875}$	0.3507	0.1128	0.3056	0.3902	0.2537	0.4253
$\mathfrak{K}^{\mathfrak{a}_{t(2)}}$							
$\mathfrak{K}^{\mathfrak{a}_{t(1)}\prime}$	$\mathfrak{L}_{0.2222}$	0.2239	0.3247	0.1613	0.2778	0.4023	0.2239
$\mathfrak{K}^{\mathfrak{a}_{t(2)}\prime}$	$\mathfrak{L}_{0.3333}$	0.2313	0.3312	0.1694	0.2847	0.4081	0.2313
$\mathfrak{K}^{\mathfrak{a}_{t(3)}\prime}$	$\mathfrak{L}_{0.5556}$	0.2388	0.3377	0.1774	0.2917	0.4138	0.2388
$\mathfrak{K}^{\mathfrak{a}_{t(4)}\prime}$	$\mathfrak{L}_{0.7778}$	0.2463	0.3442	0.1855	0.2986	0.4195	0.2463
$\mathfrak{K}^{\mathfrak{a}_{t(5)}\prime}$	$\mathfrak{L}_{0.8889}$	0.2537	0.3507	0.1936	0.3056	0.4253	0.2537
$\mathfrak{K}^{\mathfrak{a}_{t(3)}}$							
$\mathfrak{K}^{\mathfrak{a}_{t(1)}\prime}$	$\mathfrak{L}_{0.1111}$	0.2239	0.3659	0.0877	0.3247	0.4023	0.2778
$\mathfrak{K}^{\mathfrak{a}_{t(2)}\prime}$	$\mathfrak{L}_{0.3333}$	0.2313	0.3720	0.0965	0.3312	0.4081	0.2847
$\mathfrak{K}^{\mathfrak{a}_{t(3)}\prime}$	$\mathfrak{L}_{0.5556}$	0.2388	0.3781	0.1053	0.3377	0.4138	0.2917
$\mathfrak{K}^{\mathfrak{a}_{t(4)}\prime}$	$\mathfrak{L}_{0.6667}$	0.2463	0.3842	0.1140	0.3442	0.4195	0.2986
$\mathfrak{K}^{\mathfrak{a}_{t(5)}\prime}$	$\mathfrak{L}_{0.8889}$	0.2537	0.3902	0.1228	0.3507	0.4253	0.3056
$\mathfrak{K}^{\mathfrak{a}_{t(4)}}$							
$\mathfrak{K}^{\mathfrak{a}_{t(1)}\prime}$	$\mathfrak{L}_{0.3333}$	0.1613	0.2778	0.2239	0.3247	0.3659	0.2778
$\mathfrak{K}^{\mathfrak{a}_{t(2)}\prime}$	$\mathfrak{L}_{0.4444}$	0.1694	0.2847	0.2313	0.3312	0.3720	0.2847
$\mathfrak{K}^{\mathfrak{a}_{t(3)}\prime}$	$\mathfrak{L}_{0.5556}$	0.1774	0.2917	0.2388	0.3377	0.3781	0.2917
$\mathfrak{K}^{\mathfrak{a}_{t(4)}\prime}$	$\mathfrak{L}_{0.7778}$	0.1855	0.2986	0.2463	0.3442	0.3842	0.2986
$\mathfrak{K}^{\mathfrak{a}_{t(5)}\prime}$	$\mathfrak{L}_{0.8889}$	0.1936	0.3056	0.2537	0.3507	0.3902	0.3056
$\mathfrak{K}^{\mathfrak{a}_{t(5)}}$							
$\mathfrak{K}^{\mathfrak{a}_{t(1)}\prime}$	$\mathfrak{L}_{0.2222}$	0.0877	0.3659	0.3247	0.4348	0.4023	0.3659
$\mathfrak{K}^{\mathfrak{a}_{t(2)}\prime}$	$\mathfrak{L}_{0.3333}$	0.0965	0.3720	0.3312	0.4402	0.4081	0.3720
$\mathfrak{K}^{\mathfrak{a}_{t(3)}\prime}$	$\mathfrak{L}_{0.4444}$	0.1053	0.3781	0.3377	0.4457	0.4138	0.3781
$\mathfrak{K}^{\mathfrak{a}_{t(4)}\prime}$	$\mathfrak{L}_{0.6667}$	0.1140	0.3842	0.3442	0.4511	0.4195	0.3842
$\mathfrak{K}^{\mathfrak{a}_{t(5)}\prime}$	$\mathfrak{L}_{0.8889}$	0.1228	0.3902	0.3507	0.4565	0.4253	0.3902

Step 3: Determine the weighted sum model and weighted product model results using the data located in Table 6. Table 7 presents the weighted sum model and weighted product model results.

Table 7: Aggregated weighted sum and product results.

WASPAS	$\mathfrak{K}^{\mathbb{Q}_1^i}$	$\mathfrak{K}^{\mathbb{Q}_2^i}$
$\mathfrak{K}^{\mathfrak{a}_{t(1)}}$	$\begin{pmatrix}\mathfrak{L}_{0.5392},\\(0.3390,0.1070,0.2931),\\(0.3793,0.2403,0.4150)\end{pmatrix}$	$\begin{pmatrix}\mathfrak{L}_{0.4628},\\(0.3389,0.1065,0.2930),\\(0.3792,0.2402,0.4149)\end{pmatrix}$
$\mathfrak{K}^{\mathfrak{a}_{t(2)}}$	$\begin{pmatrix}\mathfrak{L}_{0.5902},\\(0.2403,0.3390,0.1791),\\(0.2931,0.4150,0.2403)\end{pmatrix}$	$\begin{pmatrix}\mathfrak{L}_{0.5402},\\(0.2402,0.3389,0.1788),\\(0.2930,0.4149,0.2402)\end{pmatrix}$
$\mathfrak{K}^{\mathfrak{a}_{t(3)}}$	$\begin{pmatrix}\mathfrak{L}_{0.5569},\\(0.2403,0.3793,0.1070),\\(0.3390,0.4150,0.2931)\end{pmatrix}$	$\begin{pmatrix}\mathfrak{L}_{0.4895},\\(0.2402,0.3792,0.1065),\\(0.3389,0.4149,0.2930)\end{pmatrix}$
$\mathfrak{K}^{\mathfrak{a}_{t(4)}}$	$\begin{pmatrix}\mathfrak{L}_{0.6231},\\(0.1791,0.2931,0.2403),\\(0.3390,0.3793,0.2931)\end{pmatrix}$	$\begin{pmatrix}\mathfrak{L}_{0.5947},\\(0.1788,0.2930,0.2402),\\(0.3389,0.3792,0.2930)\end{pmatrix}$
$\mathfrak{K}^{\mathfrak{a}_{t(5)}}$	$\begin{pmatrix}\mathfrak{L}_{0.5346},\\(0.1070,0.3793,0.3390),\\(0.4468,0.4150,0.3793)\end{pmatrix}$	$\begin{pmatrix}\mathfrak{L}_{0.4897},\\(0.1065,0.3792,0.3389),\\(0.4467,0.4149,0.3792)\end{pmatrix}$

Step 4: Calculate the aggregated WASPAS score from the data located in Table 7. Table 8 presents the final WASPAS score, where different values of λ.

Table 8: WASPAS score of the alternatives.

WASPAS	$\mathfrak{K}^{\mathfrak{a}_{t(1)}}$	$\mathfrak{K}^{\mathfrak{a}_{t(2)}}$	$\mathfrak{K}^{\mathfrak{a}_{t(3)}}$	$\mathfrak{K}^{\mathfrak{a}_{t(4)}}$	$\mathfrak{K}^{\mathfrak{a}_{t(5)}}$
$\lambda = 0.1$	0.2144	0.2097	0.2153	0.2136	0.2478
$\lambda = 0.3$	0.2150	0.2101	0.2159	0.2138	0.2482
$\lambda = 0.5$	0.2156	0.2105	0.2164	0.2140	0.2485
$\lambda = 0.7$	0.2162	0.2109	0.2169	0.2143	0.2489
$\lambda = 0.9$	0.2168	0.2113	0.2174	0.2145	0.2492

Step 5: Rank each alternative according to the score value aggregated in Table 8. Table 9 presents the final ranking result of the alternatives.

Table 9: WASPAS score of the alternatives.

WASPAS	***Ranking***
$\lambda = 0.1$	$\mathfrak{K}^{\mathfrak{a}_{t(5)}} > \mathfrak{K}^{\mathfrak{a}_{t(3)}} > \mathfrak{K}^{\mathfrak{a}_{t(1)}} > \mathfrak{K}^{\mathfrak{a}_{t(4)}} > \mathfrak{K}^{\mathfrak{a}_{t(2)}}$
$\lambda = 0.3$	$\mathfrak{K}^{\mathfrak{a}_{t(5)}} > \mathfrak{K}^{\mathfrak{a}_{t(3)}} > \mathfrak{K}^{\mathfrak{a}_{t(1)}} > \mathfrak{K}^{\mathfrak{a}_{t(4)}} > \mathfrak{K}^{\mathfrak{a}_{t(2)}}$
$\lambda = 0.5$	$\mathfrak{K}^{\mathfrak{a}_{t(5)}} > \mathfrak{K}^{\mathfrak{a}_{t(3)}} > \mathfrak{K}^{\mathfrak{a}_{t(1)}} > \mathfrak{K}^{\mathfrak{a}_{t(4)}} > \mathfrak{K}^{\mathfrak{a}_{t(2)}}$
$\lambda = 0.7$	$\mathfrak{K}^{\mathfrak{a}_{t(5)}} > \mathfrak{K}^{\mathfrak{a}_{t(3)}} > \mathfrak{K}^{\mathfrak{a}_{t(1)}} > \mathfrak{K}^{\mathfrak{a}_{t(4)}} > \mathfrak{K}^{\mathfrak{a}_{t(2)}}$
$\lambda = 0.9$	$\mathfrak{K}^{\mathfrak{a}_{t(5)}} > \mathfrak{K}^{\mathfrak{a}_{t(3)}} > \mathfrak{K}^{\mathfrak{a}_{t(1)}} > \mathfrak{K}^{\mathfrak{a}_{t(4)}} > \mathfrak{K}^{\mathfrak{a}_{t(2)}}$

The ranking result aggregated in Table 9 shows that by operating the proposed WASPAS approach on the expert information expressed in Table 5, we get $\mathfrak{K}^{\mathfrak{a}_{t(5)}}$ as the best possible alternative from the selected set of alternatives based on the given expert judgment. It shows that Flood Early Warning Systems is the most effective and reliable warning system for Natural Disaster Management among the selected ones based on the expert judgment.

f. Comparative Analysis

This section is designed to evaluate and compare the results of the proposed approach with different current ideas. It helps to provide information about the strengths and weaknesses of the proposed approach. It also shows the accuracy, reliability, and effectiveness of the proposed work. So, here in the study, we select some existing ideas for comparative analysis, such as Akram et al. [40] proposed the novel idea of m-polar fuzzy based Dombi aggregation operators and discussed their applications in decision making environment. Maity and Pal [41] constructed a modified idea of Dombi power aggregation operators under an m-polar fuzzy environment for MADM problems. Waseem et al. [42] presented the extended framework of Hamacher aggregation operators based on m-polar fuzzy data and extended its applications to the climate change domain. Deveci et al. [43] presented a modified Hamacher aggregation operator based on fuzzy information for the evaluation of climate change. Ye [45] constructed a novel idea of Frank aggregation operators based on fuzzy information and discussed their applications in a higher education environment. Chakraborty and Zavadskas [46] invented a novel WASPAS approach and extended its application to manufacturing decision making. Turskis et al. [47] constructed an integrated idea of the Analytic Hierarchy Process and the WASPAS model based on fuzzy information for the selection construction site. Ahmmad et al. [48] invented a novel MABAC model based on fuzzy rough information for disability support systems. Kizielewicz and Baczkiewicz [49] combined the evaluation results of different MADM models based on fuzzy information for the housing selection problem. Akram et al. [50] constructed a novel hybrid idea by integrating the PROMETHEE approach with the Analytic Hierarchy Process based on m-polar fuzzy information for group decision making. The detailed results of the proposed model and current ideas are explained in Table 10 below.

Table 10: Comparative analysis.

Ideas		***Scores***					***Rankings***
Akram et al. [40]		*NILL*					*NILL*
Maity and Pal [41]		*NILL*					*NILL*
Waseem et al. [42]		*NILL*					*NILL*
Deveci et al. [43]		*NILL*					*NILL*
Ye [45]		*NILL*					*NILL*
Chakraborty and Zavadskas [46]		*NILL*					*NILL*
Turskis et al. [47]		*NILL*					*NILL*
Ahmmad et al. [48]		*NILL*					*NILL*
Kizielewicz and Baczkiewicz [49]		*NILL*					*NILL*
Akram et al. [50]		*NILL*					*NILL*
WASPAS Model	$\lambda = 0.5$	$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(1)}}$ 0.2156	$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(2)}}$ 0.2105	$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(3)}}$ 0.2164	$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(4)}}$ 0.2140	$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(5)}}$ 0.2485	$\mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(5)}} > \mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(3)}} > \mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(1)}} > \mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(4)}} > \mathfrak{K}^{\mathbb{a}_{\mathfrak{t}(2)}}$

In Table 10, it is clearly shown that the current ideas are limited to handling the expert information located in Table 5, while the proposed idea effectively evaluates and ranks them.

The following are the reasons why these ideas are limited in handling the given expert information.

1. The aggregation approaches presented by Akram et al. [40], Maity and Pal [41], Waseem et al. [42], Deveci et al. [43], and Ye [45] are based on limited fuzzy extension, while the given expert judgment is in an advanced and complex form. That is why all these aggregation ideas are limited to handling such advanced information.
2. The MADM approaches invented by Chakraborty and Zavadskas [46], Turskis et al. [47], Ahmmad et al. [48], Kizielewicz and Baczkiewicz [49], and Akram et al. [50] are also based on limited fuzzy extensions, while the expressed judgment located in Table 5 are in advances from. So, because of it, these MADM approaches are also limited to evaluating and ranking it.
3. The proposed WASPAS approach is based on m-PFRL information, which is an advanced and generalized form of the traditional WASPAS approach. So, it effectively evaluates and ranks the given expert information located in Table 5, and provides a clear final result, which helps the experts to easily get the best one.

Conclusion

This research work provides an effective and reliable framework for the expert to express their judgments in a generalized way. It enhanced the decision-making approaches and the evaluation mechanism of the alternatives.

In this article, we constructed the following ideas.

1. We constructed the novel idea of the m-PFRL set and defined its fundamental properties.
2. We extended the existing idea of Frank aggregation operators to the m-PFRL environment.
3. We generalized the existing notation of the WASPAS approach and invented a novel WASPAS approach based on m-PFRL information.
4. We addressed a real-world problem of AI-based early warning systems for natural disaster management using the proposed idea.
5. We compared the results of the proposed idea with current related ideas to check its accuracy and reliability.

This research work enables the decision makers to handle complex and uncertain real-world decision-making problems. Despite these limitations, there are also some limitations for this research work, such as it cannot effectively evaluate and rank expert information expressed in m-PFRL, but if the expert expresses their preferences in an uncertain or hesitant form, then this approach will not be able to handle them.

In the future, our target is to extend the existing ideas of different aggregation operators, such as Yager aggregation operators, Dombi aggregation operators, and Einstein aggregation operators, to the m-PFRL domain. Also, our target is to extend the idea of different MADM models, such as MABAC, CoCoSo, MABAC-TOPSIS, and TOPSIS, to the m-PFRL domain.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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